The Reese T. Prosser Mathematics Lecture Series Presents

A Physical Approach to Mathematics

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Wednesday October 12, 2022 ∾ 6:00–7:00 pm ∾ Haldeman 041

How many of us were told in school that integral calculus started with Archimedes about 2,400 years ago when he computed volumes using *thought experiments* with balance scales? I certainly wasn't. Archimedes's integral calculus is one of the greatest mathematical achievements of all time, very simple to describe, and yet is almost never mentioned in our courses.

The Archimedean method of using physical thought experiments yields strikingly simple solutions of quite a few mathematical problems. The extent of this simplicity was a complete revelation to me as I kept discovering (or learning about) new "Archimedean" solutions of math problems. These problems appear at all levels, from elementary (e.g. the Pythagorean Theorem) to modern mathematics. Moreover, physical reasoning has led to new mathematical discoveries.

I will describe a few items from the list below, letting the audience make choices. No background beyond high school is required for any of these items.

1. How does the impossibility of a perpetual motion machine imply the Pythagorean theorem and much more (see item 2).
2. An instantaneous proof of the formula cos(x-y)=cos x cos y + sin x sin y using the impossibility of a perpetual motion machine.
3. Proving the Pythagorean theorem by lifting weights.
4. Solving some Min-max problems with no algebra and no calculus. Some examples: What is the fastest way for a lifeguard to reach a drowning victim? How to find the least length of fence needed to bound a rectangular area along the river? For a tin can of given volume, what proportions are most economical?, etc., etc.
5. Finding roots of a polynomial by flotation.
6. Altitudes in a triangle are concurrent: a proof using torque.
7. The geometric mean never exceeds the arithmetic mean: proofs using springs; using electric circuits; using water.

8.Discovering mathematical inequalities using electric circuits.
9. A water-based "discovery" and proof of the Cauchy-Schwartz inequality.
10. Euler's polyhedral formula E-V+F=2 proved using electricity.
11. What is a symplectic map and how is it related to the Archimedean seesaw?
12. Bernoulli's brachistochrone: the simplest solution using a mechanical analog computer.
13. Why does the billiard ball shot from one focus of an elliptical table get reflected to the other focus?
14. How to deform a planar region without distortion on the small scale?
15. What does a telescope magnification have to do with the seesaw?