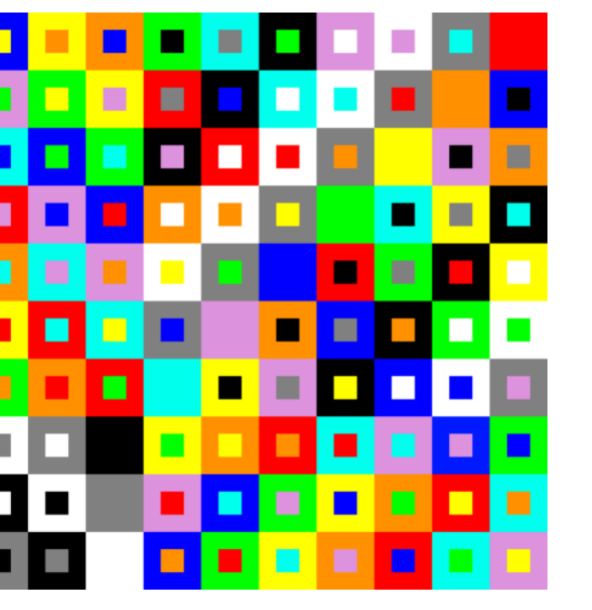


Model for a Closed Life Support System with Fitness Model and Standard Replicator Equation

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Abstract

As NASA, Elon Musk, and other space exploration groups set their sights on a manned mission to Mars, the ability to feed and sustain humans for extended periods of time outside of low-Earth orbit is critical. Life support systems need to be strong, reliable, and will need to be able to effectively recycle nutrients, water, and gases.¹ Current systems, such as the Environmental Control and Life Support (ECLS), are not fully closed and need to be habitually re-supplied in order to function continually.² As such, self-sustaining biological systems, including intensive agriculture, are active areas of research for NASA and private aerospace companies.³

Our project attempts to model and optimize the biomass production systems of a closed life support system that could be used in space. By pairing different plant types within first a logistic growth model and a replicator equation model, we show that food production can be easily controlled in order to consistently support humans aboard the spacecraft, and that the use of gases and nutrients can be balanced to maintain natural equilibria.

Methods

While current models such as the ECLS require frequent resource inputs to maintain continuous function, the goal of our design was to make our model self-sufficient.⁴ In such a model, once a balance is attained, there is no need for either import or export of additional resources. Of course, the system must also maintain biomass production above a certain level necessary to maintain human life.⁵ Bacteria subsist alongside the plants, and provide the gases necessary for their survival, while the plants compete for available resources.

Logistic Growth Model

As a first approximation of the behavior of plant species in a closed system with a fixed maximum carrying capacity for multiple populations, we consider a modified logistic growth model, where the carrying capacities for a given population depend on pairwise interactions with the other species involved.^{6,11} We consider the consumption of each species to be directly proportional to the mass of the species. The derivatives of the population values for each species are given to be the following:

$$(1) \begin{aligned} \dot{P}_A &= P_A \left(1 - \frac{P_A}{k_A} - \frac{P_B}{k_B} - \frac{P_C}{k_C}\right) \\ \dot{P}_B &= P_B \left(1 - \frac{P_A}{k_A} - \frac{P_B}{k_B} - \frac{P_C}{k_C}\right) \\ \dot{P}_C &= P_C \left(1 - \frac{P_A}{k_A} - \frac{P_B}{k_B} - \frac{P_C}{k_C}\right) \end{aligned}$$

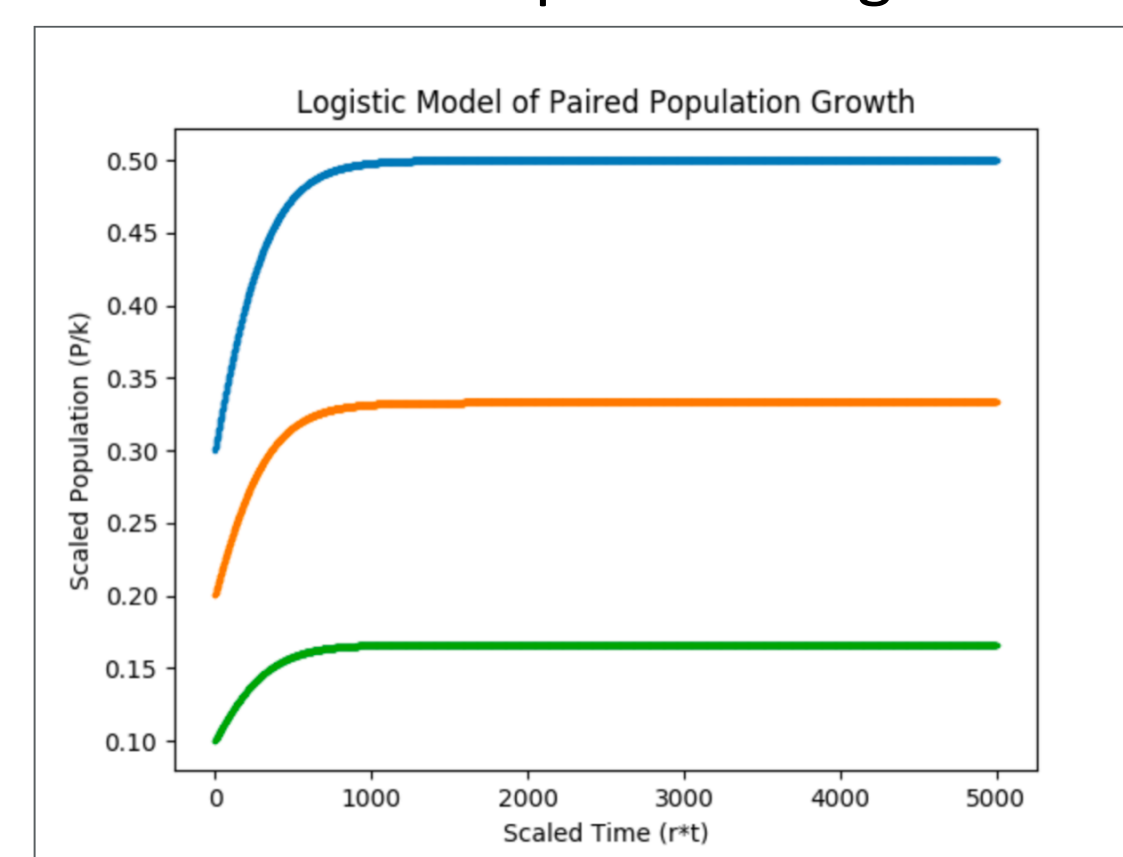


Fig. 1: Growth of populations under a paired logistic model.

Intuitively, the populations will change until they achieve a sum equal to the total carrying capacity of the system, in this case scaled to be 1. We can test the stability of the equilibrium for a two species system in which the populations sum to 1, meaning that they are at maximum capacity. We find that the Jacobian has one negative eigenvalue and one zero eigenvalue, making our analysis inconclusive. However, considering a perturbation of the population system about an equilibrium point, changing one of the populations by a small amount δ , we find that both negative and positive changes will decay back to that point. Let the k values be scaled to one for each of the species. Using $P_B=1-P_A$:

$$\dot{P}_A(\delta + P_A, P_B, P_C) = (P_A + \delta)(1 - (P_A + \delta) - (1 - P_A)) = -\delta(P_A + \delta)$$

The derivative after a deviation from an equilibrium point will thus always be in the opposite direction of the change that was made, restoring the state of the system. The equilibria are thus stable. We can establish by inspection that the system of three populations will have zero derivatives where all three of the populations sum to one, meaning they lie in the plane $1=P_A+P_B+P_C$. A more interesting case to consider is that of partial overlap in resource consumption, where each species only conflicts with one other.

The system of equations as given in (3) then has nullclines in the form of intersecting planes, identified in the three equations in (4).

$$(3) \begin{aligned} \dot{P}_A &= P_A \left(1 - \frac{P_A}{k_A} - \frac{P_B}{k_B}\right) \\ \dot{P}_B &= P_B \left(1 - \frac{P_B}{k_B} - \frac{P_C}{k_C}\right) \\ \dot{P}_C &= P_C \left(1 - \frac{P_A}{k_A} - \frac{P_C}{k_C}\right) \end{aligned} \quad (4) \begin{aligned} 1 &= \frac{P_A}{k_A} + \frac{P_B}{k_B} \\ 1 &= \frac{P_B}{k_B} + \frac{P_C}{k_C} \\ 1 &= \frac{P_A}{k_A} + \frac{P_C}{k_C} \end{aligned}$$

This system has potential equilibria where those nullclines intersect, which will be at points where one population is at its carrying capacity k and the other 2 are zero, or where each is at half of its carrying capacity (This point is evident in fig. 2). Similar plots for other pairs of dimensions show that this point is stable. We can compare our results from this model with those of a replicator equation model.

Fitness Model and Standard Replicator Equation

We can use the replicator equation from evolutionary game theory to understand the dynamics of interacting populations all competing for a common resource.⁸ We create a "consumption profile" for each species consisting of a vector of length m , where there are m resources of interest in the system. Each element of the vector for a given species indicates the rate at which they consume that particular resource. Using this model, we can consider the overlap between the consumption profiles of two species to be the dot product of the vectors for each population. This dot product serves as a metric for the cost to each of the species of sharing space with the other. The replicator equation that we begin with is given in the following form, where \bar{f} indicates the the average fitness of the species involved⁹:

$$(5) \dot{P}_i = P_i(f_i - \bar{f}), \text{ where } \bar{f} = \frac{f_1 + \dots + f_n}{n}$$

If we consider as an example a system of three populations, a , b , and c , with given consumption vectors $C_A = [1, .9, .8]$, $C_B = [.8, 1, .9]$, and $C_C = [-1, -1, -1]$, we can establish the null-clines of the system and then take their intersection to find potential equilibria. For internal points we can attempt to find the eigenvalues of the Jacobian at any potential equilibria to understand their stability. In this particular case, we find a potential equilibrium forms at very low P_B and nearly equal P_A and P_C . The Jacobian has two negative eigenvalues and one zero eigenvalue, suggesting that the point may be stable but we cannot be sure. Plotting the behavior in a simplex plot,⁸ we can see that it is indeed stable.

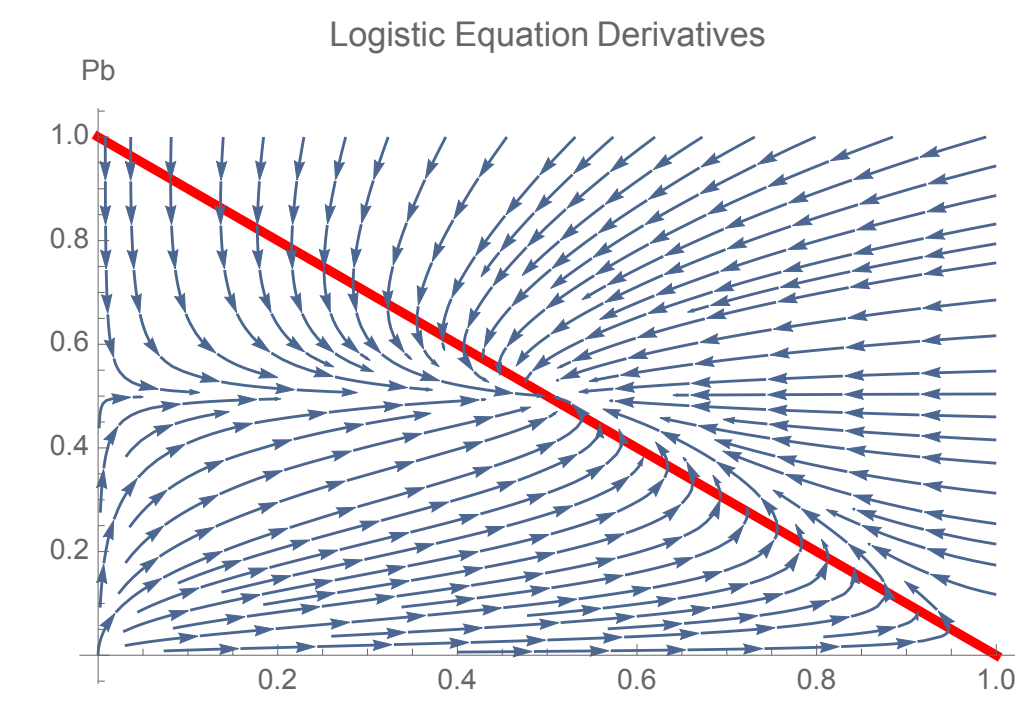


Fig. 2: Nullcline and Streamplot at $P_C=.5$.

As a proxy for internal behavior we can consider the pairwise interactions along the "edges" of our solution domain, where one of the populations is zero. Let population B be equal to zero. Then we consider the pairwise interaction of populations A and C. Along the edge we have the derivative values described by (6):

$$(6) \dot{P}_A = \frac{P_A}{3} (P_A(k_1 - k_2) + (k_2)), \text{ where}$$

$$k_1 = 2M_{AA} - M_{BA},$$

$$k_2 = M_{AB} - M_{BB},$$

The equilibrium point, found where $P_A = k_2/(k_2 - k_1)$, will be stable if $(k_1 - k_2)$ is negative and k_2 is positive, which can be established via an analysis of the second derivative at the point in question. The derivative of P_C is equal to the negative of the derivative of P_A in this case, as $P_C=1-P_A$ along the edge. In Fig. 4, for instance, the above analysis leads to the conclusion that there is a stable equilibrium at $P_B=.5$, in keeping with what is shown in the Simplex plot.

We can consider instead a case with less overlap in the nutrient requirements of the species involved: $C_A = [1, .5, .1]$, $C_B = [1, .5, 1]$, and $C_C = [-1, -1, -1]$. We now find only one equilibrium point, with $P_A=0.326$, $P_B=0.326$, $P_C = 0.348$. However, we find that the Jacobian has two negative eigenvalues and one zero eigenvalue and is thus inconclusive as a measure of stability. Plotting the behavior of the derivatives around this point, we find that it is in fact a stable degenerate node.

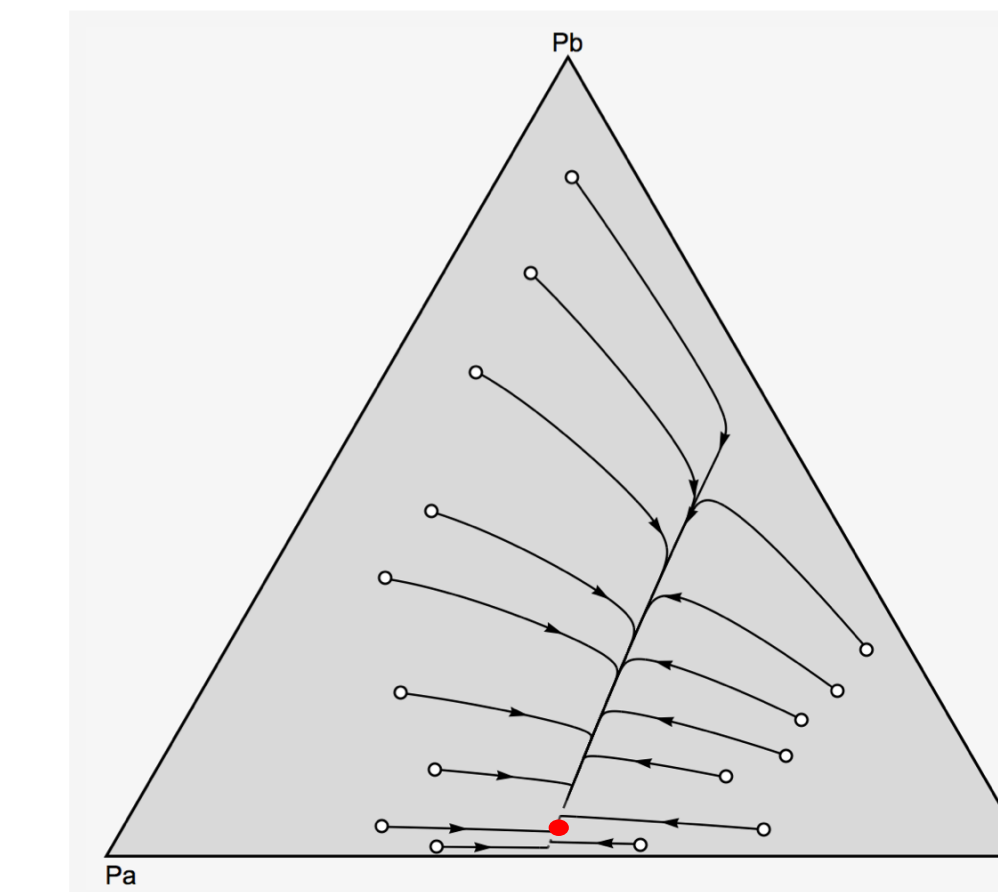


Fig. 3: $C_1 = [1, .9, .8]$, $C_2 = [.8, 1, .9]$, $C_3 = [-1, -1, -1]$

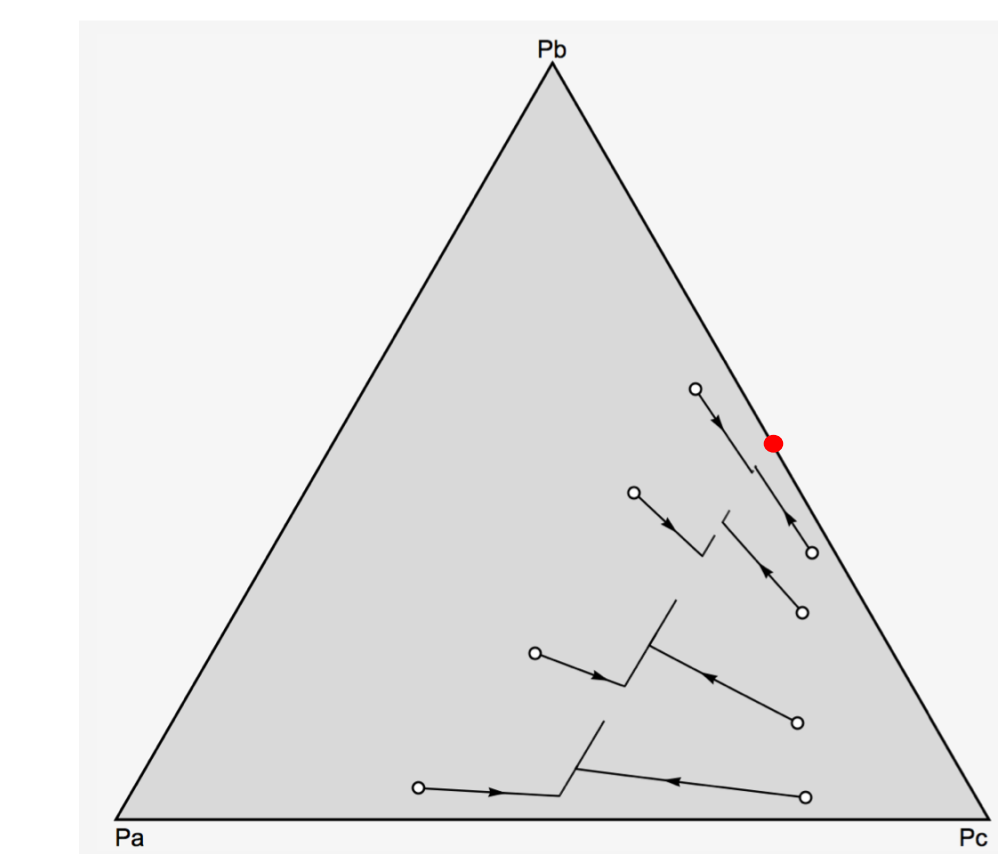


Fig. 4: $C_1 = [1, 1, .9]$, $C_2 = [1, 1, 1]$, $C_3 = [-1, -1, -1]$

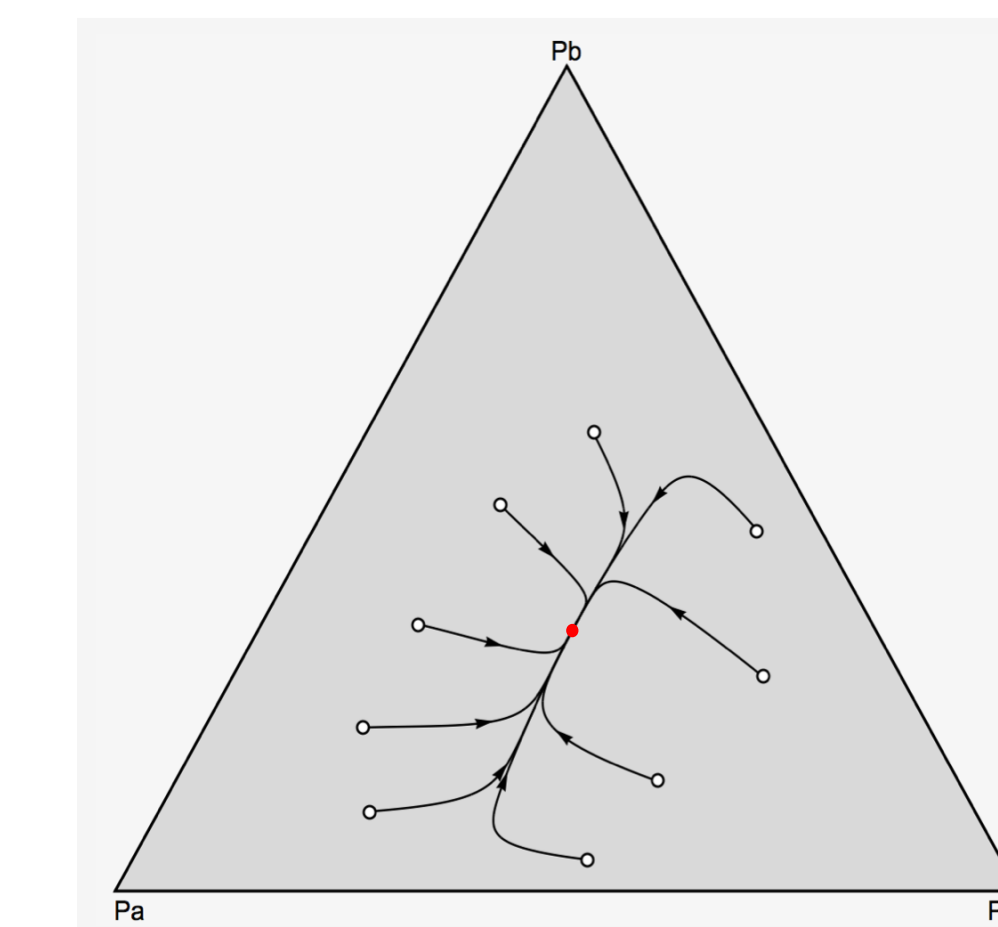


Fig. 5: $C_A = [1, .5, .1]$, $C_B = [1, .5, 1]$, $C_C = [-1, -1, -1]$.

Considering the nullclines of different combinations of consumption profiles, we find that it is possible to create whole lines of solutions. If we allow for three species where one produces something that other two need (their consumption profiles thus looking like $C_A = [1, 1, -1]$, $C_B = [1, 1, -1]$, and $C_C = [-1, -1, 1]$), we find that any combination of P_A and P_B summing to .5 is permissible, but P_C will be fixed at .5 in order to support the other two populations.

We can also find solutions in which the populations of two of the species depend on the population of one of them, as in the profile combination $C_A = [1, 1, 1]$, $C_B = [1, 1, -1]$, and $C_C = [-1, -1, 1]$. The line of solutions runs in this case from $P_A = .5$ and $P_B = 0$ to $P_A = 0$ and $P_B = .667$. This is intriguing for the purpose of space travel in that it may allow for the control of the system of three species via the manipulation of one. There are potentially energetically advantageous ways of doing this, for instance by varying heat or light access to a population of bacteria or plants.

Discussion and Conclusion

While in space, any life support system must perform predictably and reliably. Both of the models considered demonstrate the viability of a system of three interdependent species and provide methods for the prediction of stable equilibria in that system as a function of the overlap in resource profiles for each species. With respect to a functioning life-support system, each stable equilibrium represents a different combination of biomass types that could be maintained with minimal additional energy on the part of the crew. Of particular note are the equilibria points that may be influenced easily to arrive at a desired point along a curve. We showed that in the case of lines of admissible solutions, a user of the system may easily and predictably change the production of two plants A and B by increasing or decreasing the amount of population C, which might for instance be a population of bacteria producing carbon dioxide and other nutrients. In this manner, the mission's food production may be both planned ahead of time and controlled in transit, all without concern for hauling large amounts of food into orbit, which has historically represented a large portion of non-propulsion mass.¹⁰ This analysis can be further expanded to more plants and more bacteria for larger variety, with different equilibria corresponding to each combination. Although our analysis is limited to the biomass production system, further research may expand upon and introduce additional sub-systems from waste disposal to gas recycling and water processing. We have found that the logistic model in this case does provide an intuitive model for the end state of the system but does not capture as completely the idea of competition for a common resource based on overlapping biological niches. The comparatively more complicated replicator dynamics model provides a more natural implementation of this interaction.

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