Varying Measles Vaccination Rates in a Theoretical Prison Population

Addison Green, Erika Hernandez, Kenna Van Steyn, Dr. Dorothy Wallace
Dartmouth College Department of Mathematics

Abstract

In order to understand how varying vaccination rates affects the transmission of measles in a theoretical constant population of prisoners, the traditional SEIR model was adapted to account for the unique timeline of measles symptoms/contagiousness as well as for MMR vaccination rates. Drawing from research literature for parameters, the relative timeline for symptom onset was accounted for and the vaccination rates were varied on different runs in Big Green Differential Equation Machine software. Increasing vaccination rates while keeping all other parameters constant resulted in a decrease in the number of susceptible individuals, and thus, a lower overall transmission throughout the population. A 95% population vaccination percentage, the herd immunity threshold for measles, was the most effective means to control the spread of measles in a small prison environment.

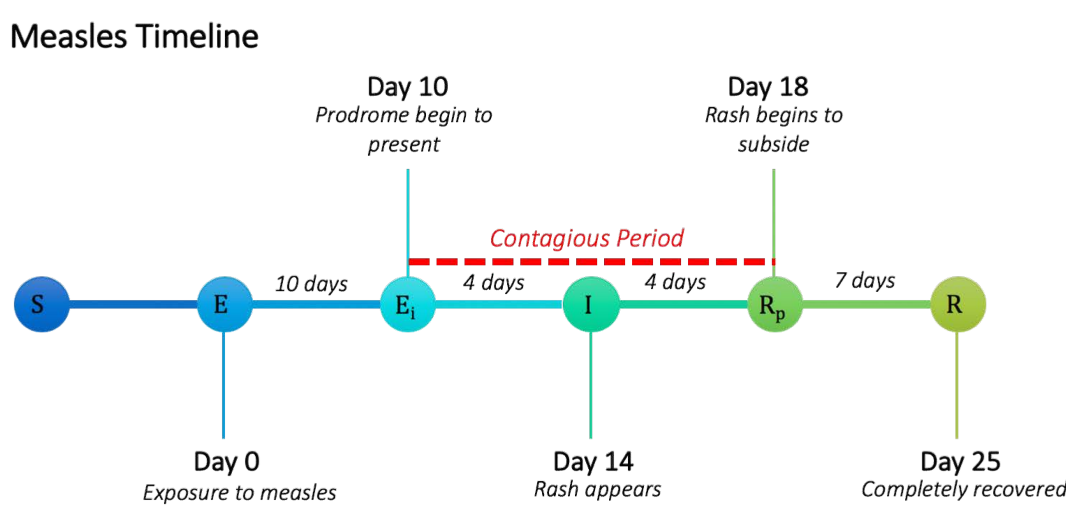
Methodology

1) Produced theoretical values for each population
2) Collected values for the parameters from literature
3) Formulated equations for the rate of change relative to time for S, E, E_i, I, R_p, and R
4) Inputted equations, initial conditions, and parameters into Big Green Differential Equation Machine software to yield graphs of population over time
5) Conducted multiple runs with different percentages of the population vaccinated

Introduction

- Measles is an acute viral infectious disease that is spread through coughing and sneezing. It is highly contagious.
- Up to 90% of people close to the individual who are not immune to measles will become infected with the disease
- Once a person is infected with measles once, their body develops life-long immunity via antibodies.
- Symptoms include: fever, dry cough, runny nose, sore throat, conjunctivitis, skin rash, and Koplik's spots (white spots lining mouth)
- MMR vaccine, used since the 1960s, is given in two doses: first dose administered between the ages of 12 to 15 months after birth and the second dose between the ages of 4 to 6 years old
- Those who receive both doses, are effectively 99% immune to measles.
- Some people, due to religious and other personal reasons, have not vaccinated their children and/or do not get the MMR vaccine themselves
- Measles is on the rise in the past decade in the US because of 'holes' in herd immunity (95 % or more of population need to be vaccinated for population to be protected

In a prison setting, it is important to consider:
- access to vulnerable social groups from many times disadvantaged socioeconomic and educational backgrounds who may not have used national health services
- high risk of contraction (due to overpopulation, overcrowding, and lack of hygiene)
- how minimum-security prisoners are constantly in contact with rest of community



Explanation of Model

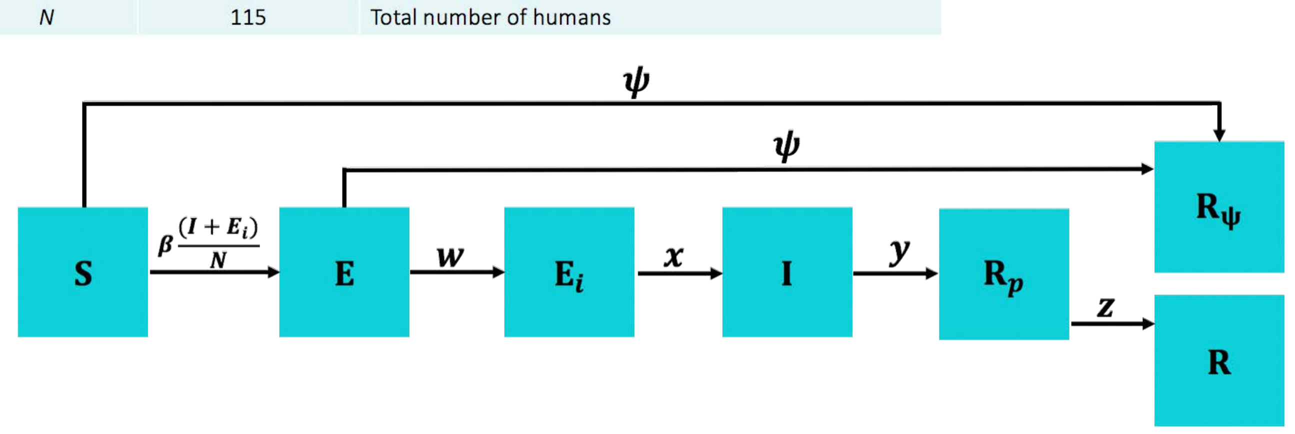
In this modified SEIR model with vaccination, a person has three possible paths:
- susceptible (S) → exposed (E) → exposed and infectious (E_i) → infectious (I) → partially recovered (R_p) → completely recovered (R).
- susceptible (S) → recovered from vaccine (R_ψ)
- exposed (E) →recovered from vaccine (R_ψ)

Aims:
- as vaccination rate increase, exhibit less infectious and more recovered people
- show how increasing vaccination rates lessens spread of measles in population

Assumptions:
- Homogenous and constant population of prisoners
- No death from measles (since it is negligible in the US)
- Standardization of the timeline of measles to a set number of days
- Vaccination rate is assumed 100% effective for the sake of simplification
- Theoretical numbers of prisoners in each compartment

Parameter	Value	Meaning *Value shown is initial condition
S	100	Number of susceptible humans*
E	5	Number of exposed but not yet contagious humans*
E_i	5	Number of exposed humans who are contagious but not yet showing rash*
I	5	Number of infected humans who are contagious*
R_p	0	Number humans who are not contagious anymore but only partially recovered*
R	0	Number of humans fully recovered from illness*
R_ψ	0	Number of humans who have been vaccinated and are thus recovered*
ψ	Varies per graph	Vaccination rate
β	0.9	Probability of infection upon contact
w	0.1	Rate at which exposed become contagious without showing rash
x	0.07143	Rate at which the exposed who are contagious become infected with rash
y	0.05556	Rate at which infected humans who are contagious become no longer contagious but are not yet completely recovered
z	0.04	Rate at which the not yet completely recovered become completely recovered
N	115	Total number of humans

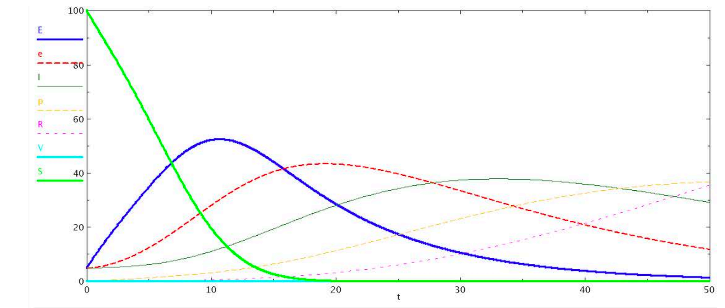
$$\frac{dS}{dt} = -\beta S\frac{(I+E_i)}{N} - \psi S$$
$$\frac{dE}{dt} = \beta S\frac{(I+E_i)}{N} - wE - \psi E$$
$$\frac{dE_i}{dt} = wE - xE_i$$
$$\frac{dI}{dt} = xE_i - yI$$
$$\frac{dR_p}{dt} = -zR_p + yI$$
$$\frac{dR}{dt} = zR_p$$
$$\frac{dR_\psi}{dt} = S\psi + E\psi$$
$$N = S + E + E_i + I + R_p + R + R_\psi$$



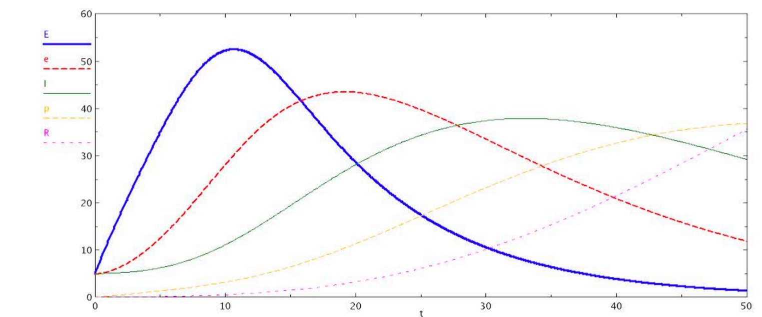
Results

Parameter	Input in Big Green
E	E
E_i	e
I	I
R_p	p
R	R
R_ψ	V
S	S

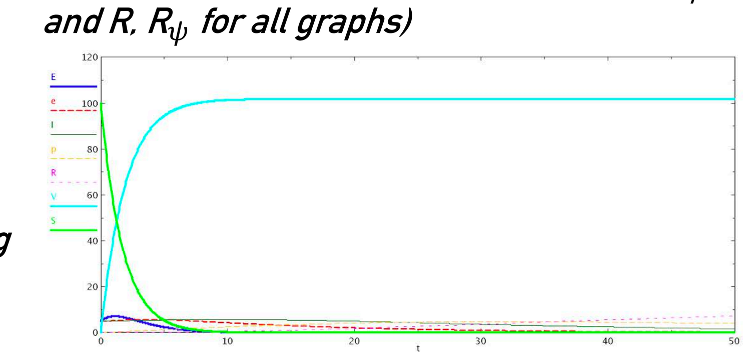
Figure: Legend showing parameters and the corresponding input into Big Green software



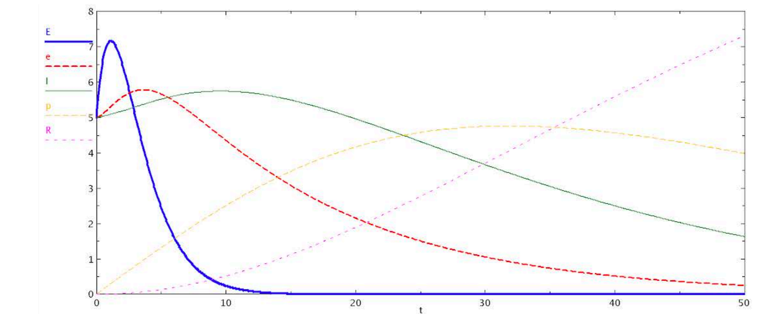
Graph of Run where Vaccination Rate=0
(Refer to table of parameters to see the values and initial conditions for S, E, E_i, I, R_p and R, R_ψ for all graphs)



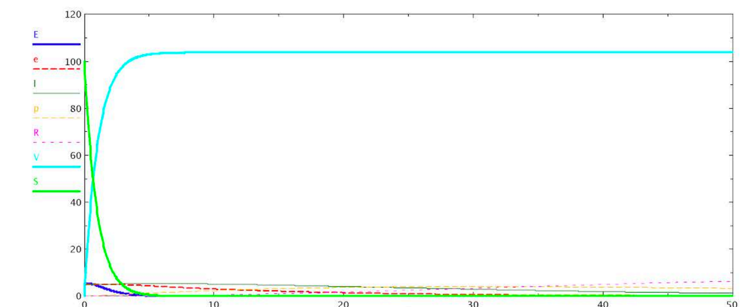
Close-up Graph of Parameters E, e, I, p, R for for run where Vaccination Rate=0



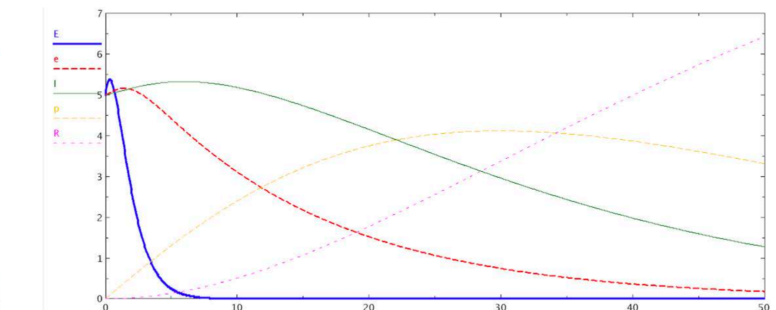
Graph of Run where Vaccination Rate=0.5



Close-up Graph of Parameters E, e, I, p, R for for run where Vaccination Rate=0.5



Graph of Run where Vaccination Rate=0.95



Close-up Graph of Parameters E, e, I, p, R for for run where Vaccination Rate=0.95

Conclusion

Increasing the vaccination rate for each run yielded a faster decay of exposed people with every increment. Due to more susceptible people being vaccinated, less people were likely to be exposed and move from one compartment to the next. When the vaccination rates were increased, the number of people who automatically became recovered as a result of the vaccination were also increased, and thus, the flow from susceptible to recovered from measles was stunted. This is a clear, effective model for the spread of measles in a closed population. Even a slight vaccination increase in vaccination rates will yield more protection to the prison population against a measles outbreak. The rate of change in recovered people drastically increases with every vaccination percentage increment. This model strongly supports that the MMR vaccination does increase protection for susceptible inmates in prison. It adds to previous models (simple SEIR) through additional compartments and a timeline specific to measles chronology, addresses the rates at which susceptible subgroups transmit the disease as well as cases wherein vaccination protects individuals.

Throughout the past few decades, large achievements have been reached regarding infectious disease control. However, measles elimination is still a pressing concern. False information (i.e. how the MMR vaccine may cause autism), religious affiliation that prevents vaccination, and lack of access to vaccination services, to list a few, are all pertinent barriers of entry for fully eliminating measles in the United States. Disease transmission, especially in a closed setting like a prison, without adequate vaccination rates places many vulnerable individuals at risk. It is thus, important to vaccinate individuals for measles to achieve herd immunity of 95%, especially in crowded places such as prisons where the same people are constantly interacting with one another. This will ensure that measles outbreaks do not occur and that vulnerable populations are adequately protected.

References

[1] Measles CDC. (n.d.). Retrieved May 8, 2019, from https://www.cdc.gov/vaccines/pubs/pinkbook/downloads/meas.pdf [2] Measles | Signs and Symptoms | CDC. (n.d.). Retrieved May 6, 2019, from https://www.cdc.gov/measles/about/signs-symptoms.html [3] Measles: Symptoms, Diagnosis, and Treatments. (2016). Retrieved May 8, 2019, from https://www.healthline.com/health/measles [4] Measles. (n.d.). Retrieved May 3, 2019, from https://www.who.int/news-room/fact-sheets/detail/measles [5] Measles and Rubella Surveillance. (2019, April 15). Retrieved May 3, 2019, from https://www.who.int/immunization/monitoring_surveillance/burden/vpd/surveillance_type/active/measles_monthlydata/en/ [6] MMR Vaccine: What You Need to Know (VIS). (n.d.). Retrieved May 4, 2019, from https://www.healthychildren.org/English/safety-prevention/immunizations/Pages/Measles-Mumps-Rubella-Vaccines-What-You-Need-to-Know.aspx [7] Measles Statistics 2019: Cases & A Map. (n.d.). Retrieved May 13, 2019, from https://www.clearvuehealth.com/b/measles-statistics/ [8] Funk, S. (2017, October 19). Critical immunity thresholds for measles elimination. Retrieved May 9, 2019, from https://www.who.int/immunization/sage/meetings/2017/october/2._target_immunity_levels_FUNK.pdf [9] Sequera, V., Valencia, S., García-Basteiro, A. L., Marco, A., & Bayas, J. M. (2015). Vaccinations in prisons: A shot in the arm for community health. Human Vaccines & Immunotherapeutics,11(11), 2615-2626. doi:10.1080/21645515.2015.1051269 [10] An SEIR Model. (n.d.). Retrieved May 13, 2019, from https://sites.me.ucsb.edu/~moehlis/APC514/tutorials/tutorial_seasonal/node4.html [11] Measles | Transmission | CDC. (n.d.). Retrieved May 13, 2019, from https://www.cdc.gov/measles/about/transmission.html [12] Sawyer, W. & Wagner, P. (March 2019). Mass Incarceration: The Whole Pie 2019. Retrieved May 13, 2019 from https://www.prisonpolicy.org/reports/pie2019.html [13] BPReid (2002). The Big Green Ordinary Differential Equation Machine. Retrieved May 5, 2019 from Dartmouth College Mathematics Department.