Evolutionary Game Theory Analysis of Asset and Loan Diversification in the Banking System

Zimehr Abbasi, Min Hur, Jessica Wang, Ryan Xu, Austin Hyun, Tate McDowell
Department of Mathematics, Dartmouth College

Abstract

Banking institutions serve as the cornerstone of modern civilization. As such, the collapse of banks has extreme ripple effects that harm all players—from corporate- to individual-level. Using Evolutionary Game Theory (EGT) principles, we aim to investigate what types of strategies minimize the risk of defaulting in the face of volatile economic landscapes. Through a series of simulated games, in which banks play the market with varying investment strategies, we find that moderate connectivity (p = 0.4) between banks and the utilization of risk-averse strategies had the best chances for long-term survival after multiple iterations of the game .

Introduction

Financial institutions such as banks play a crucial role in maintaining the U.S economy. The Great Financial Crisis of 2008 demonstrated how unmonitored risky behaviors by these financial institutions can negatively impact the economy, and recent bank collapses of Silicon Valley Bank and First Republic Bank in March 2023 have once again put the United States banking system on the forefront of many people's minds.

svb Silicon Valley Bank

First Republic Bank

Figure 1: Logos of Silicon Valley Bank and First Republic Bank

However, what if there was a way to predict how to mitigate the risk that banks take on in order to potentially prevent a financial crisis such as the SVB bank run in the future?" More specifically, what if there was a way to utilize the concepts of Evolutionary Game Theory (EGT) in order to model the strategies that banks could take on in order to minimize the risk of failure in the future? Utilizing the concepts of EGT, we designed a model to simulate the interactions that banks have with with the overall market and how differently-positioned banks react to different types of shocks. This simulation provides insight in examining which strategies banks should adopt in order to minimize their chance of failure, which protects the economy in the long run.

Notation

The following notation is adopted: Let **An×n** represent an $n \times n$ matrix. A vector is denoted by **a**, with *ai* referring to the *i*th element of **a**. When referring to the *i*th row of matrix **A**, we use the notation **Ai**,:, and when referencing the *i*th column, we use **A**:,**i**. An element located in the *i*th row and *j*th column of matrix **A** is denoted as *Ai,j*. The vector of all ones is represented by **1**, while **0** denotes the vector of all zeros. The identity matrix is denoted by **I**. Random variables are represented by capital letters, such as *X*, with their expected value and variance denoted as **E**[*X*] or *μX* and **V**[*X*] or *σX*, respectively.

Methodology

We applied EGT principles to analyze how banks evolve over time based on their chosen investment strategies. To facilitate this analysis, we conducted a simulation that simulated the interactions among banks over a defined time period. In our simulation, we represented the interactions between banks as a repeated game, wherein each bank had to follow a predetermined investment strategy designed specifically for this game. The investment strategies encompass various approaches and are listed in the "Strategies" section. Below is a closer look at how the simulation was created.

- Through the Federal Deposit Insurance Corporation (FDIC), we utilized data on assets and liabilities of 4715 banks to create a log normal distribution of bank size by assets and equities. 500 points from this distribution are randomly chosen to represent 500 banks.

Methodology Continued

- For each simulation, a bank's total assets is comprised of internal and external assets, and we create an **interconnected network of these banks via internal assets**, in which a ratio of internal assets to total assets is randomly generated using a binormal distribution.
- A bank's internal assets is distributed out to other banks. We used the **Erdos-Renyi** model to generate this bank network.
- For each turn in the simulation, bank **invests** all available external assets across 10 asset classes **based on assigned strategy** independent of other banks. Each asset class has an expected return and standard deviation. A point in the distribution is randomly chosen as the actual return.
- With the assumption that liabilities do not change, bank's assets are recalculated and checked for insolvency (when liabilities exceed assets). If insolvent, bank is removed from the network.
- Because the value of internal assets decreases when a bank goes insolvent to cover the loss of external assets, all other banks are recalculated to see if they went insolvent as a result.
- A new turn starts, and remaining banks in the network invest their external assets again. Simulation continues for 50 turns.
- For each simulation, a shock is introduced to replicate the unpredictability of the real word, and the different shocks are listed in the "Shocks" section.

Equations

$$(\mathbf{A} - \mathbf{A}^{\mathrm{T}})\mathbf{1} + \mathbf{a} - 1 > 0$$

$$f(x) = \frac{2}{1 + e^{-0.1(x+m)}} - 1 + n$$

$$a_i + \mathbf{A}_{\mathbf{i},:}\mathbf{1}$$

$$l_i + \mathbf{A}_{\mathbf{i},:}^{\mathrm{T}}\mathbf{1}$$

$$a_i + \mathbf{A}_{\mathbf{i},:}\mathbf{1} - l_i - \mathbf{A}_{\mathbf{i},:}^{\mathrm{T}}\mathbf{1}$$

$$\frac{A_{j,i}}{\mathbf{A}_{:,\mathbf{i}}\mathbf{1}} \cdot (1 - h) \cdot (d + \mathbf{A}_{i,:}\mathbf{1})$$

Strategies

0. **Highest Expected Return**: Banks will invest in the asset with the highest expected return.
1. **Lowest Volatility**: Banks will invest in the asset with the lowest volatility.
2. **Median Expected Return**: Banks will invest in the asset with the median expected return.
3. **Equally Weighted**: Banks will invest equally in all assets.
4. **Proportional to Asset Returns**: Banks will invest in assets in proportion to their expected returns.
5. **Inversely Proportional to Asset Volatilities**: Banks will invest in assets inversely proportional to their volatilities.
6. **Top Quartile Expected Return**: Banks will invest in the asset with the highest expected return in the top quartile.
7. **Lowest Quartile Volatility**: Banks will invest in the asset with the lowest volatility in the lowest quartile.
8. **Median Volatility:** Banks will invest in the asset with the median volatility

Shocks

1. **Asset Devaluation Shock:** This shock randomly selected an asset from the available pool and de-valued it by 50%.
2. **Volatility-based Asset Devaluation Shock:** In this shock, an asset was chosen with a probability proportional to its volatility. Subsequently, the selected asset was devalued by 50%.
3. **Random Bank Default Shock:** This shock involved randomly selecting a bank and causing it to default.
4. **Equity-based Bank Default Shock:** In this shock, a bank was chosen for default with a probability inversely proportional to its equity. Banks with lower equity were more susceptible to defaulting under this shock.

Results

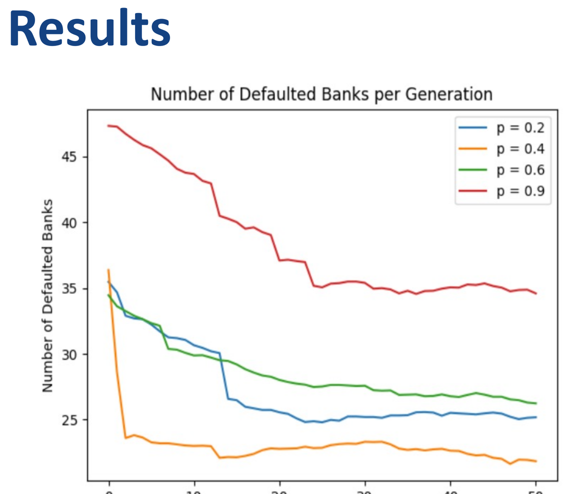

Figure 2: Default Rate for Each *p*

p	Initial Default Rate	Final Default Rate
0.2	7%	5.2%
0.4	7.4%	4.4%
0.6	6.8%	5.6%
0.9	9.6%	7%

Table 1: Initial and Final Default Rates for Each

Figure 2 and Table 1 show the results of changing the level of connectedness between banks, *p*.

We found that higher levels of connectedness between banks were shown to be correlated with higher rates of defaulting. Figures 3 through 6 show the evolutionarily stable strategy at different levels of interconnectedness. We found that, analogous to default rates, interconnectedness was also correlated to investment strategy.

Results Continued

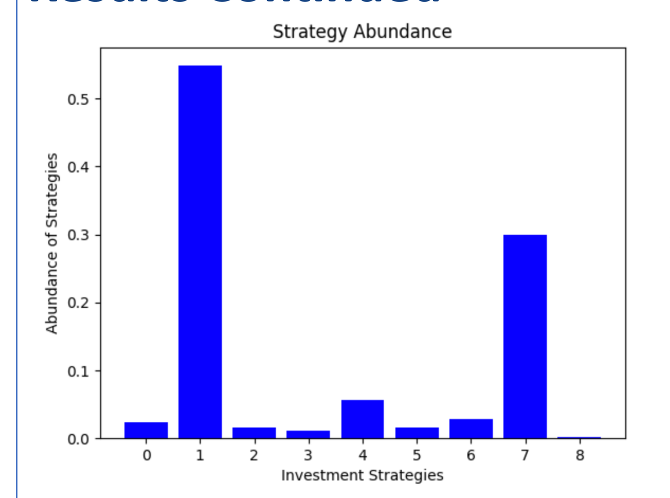

Figure 3: Evolutionary Stable Strategy for p=0.2

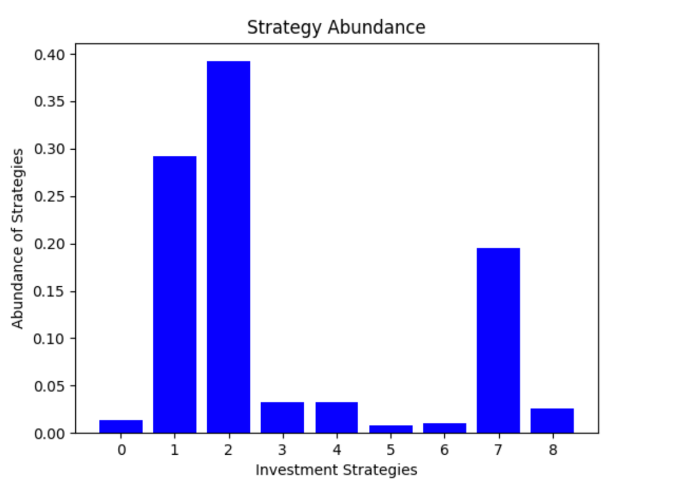

Figure 5: Evolutionary Stable Strategy for p=0.6

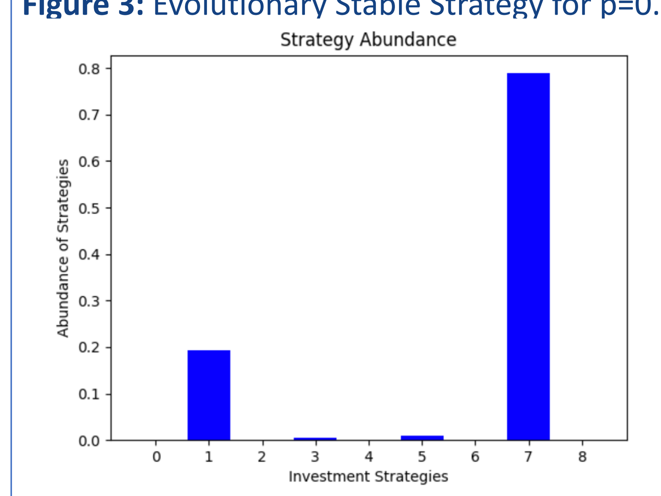

Figure 4: Evolutionary Stable Strategy for p=0.4

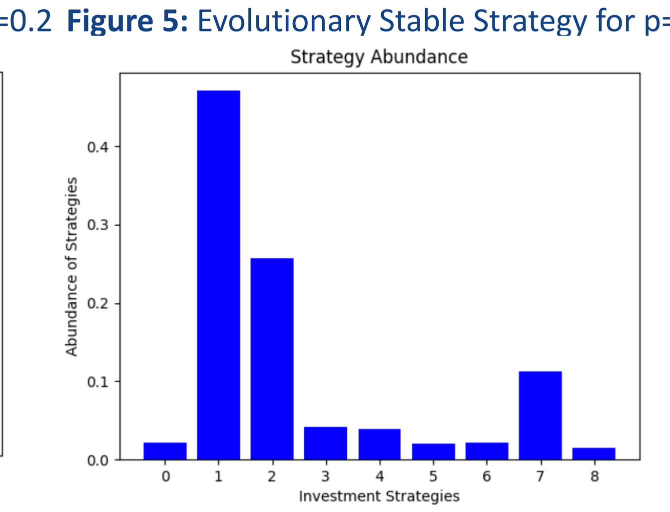

Figure 6: Evolutionary Stable Strategy for p=0.9

Discussion

We found that the level of connectedness between banks (*p*) had effects on the default rates and evolutionarily stable state strategy.

Default Rate Analysis

Our results imply that a balanced approach to connectivity is crucial for banks aiming to minimize the probability of default. These findings have implications for policymakers and regulators in promoting stability and resilience within the banking sector.

Strategy Analysis

Despite varying levels of prominence depending on bank connectedness, strategies 1, 2, and 7 emerge as generally the most evolutionarily stable. The varying dominance of strategies across different levels of connectivity highlights the importance of adaptability and risk aversion for banks seeking long-term viability in a dynamic banking system.

Conclusion

The simulations indicate that the best investment strategies for banks to prevent defaulting is to choose strategies involving less risk and more consistent returns than those with the highest possible expected returns. Although we recognize that our model makes multiple simplifying assumptions and does not perfectly capture banks' risk-taking behaviors or the unpredictability of the real world, an EGT approach provides valuable insight in observing inter-bank dynamics and banks' interactions with both internal and external shocks. Our findings hope to demonstrate that it is best for the actors in the economy to foster an environment of cooperation on the strategies that they take rather than an environment of competition. Although the higher expected returns may be attractive, it is more important to focus on the bigger picture of the collective good of our society.

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