Spectral sequences and Khovanov homology

Zachary Winkeler

Abstract

In this thesis, we will focus on two main topics; the common thread between both will be the existence of spectral sequences relating Khovanov homology to other knot invariants.

Our first topic is an invariant $\mathrm{MKh}(L)$ for links in thickened disks with multiple punctures. This invariant is different from but inspired by both the Asaeda-Pryzytycki-Sikora (APS) homology and its specialization to links in the solid torus. Our theory will be constructed from a $\mathbb{Z}^n$-filtration on the Khovanov complex, and as a result we will get various spectral sequences relating $\mathrm{MKh}(L)$ to $\mathrm{Kh}(L)$, $\mathrm{AKh}(L)$, and $\mathrm{APS}(L)$.

Our second topic is the Dowlin spectral sequence, which has E_2-page isomorphic to the reduced Khovanov homology $\overline{\mathrm{Kh}}(L)$, and which converges to the knot Floer homology $\widehat{\mathrm{HFK}}(L)$ on the E_∞-page. While it was previously known that these two pages are link invariants, we prove that every page is an invariant by defining weak maps on the underlying filtered complex which correspond to Reidemeister moves. This result is based on joint work with Samuel Tripp.